Markov Processes

Section 8.1-8.2

April 14, 2016

A coin is tossed at most five times. The tosser wins as soon as the number of heads exceeds the number of tails and loses as soon as three tails have been tossed. Use a tree diagram to compute the probability of winning.

Group Quiz

Suppose an investment banker runs the following experiment every day for a year. She observes how the price of a certain stock changes at the end of each day. The possible outcomes are 'increased', 'decreased', or 'remains unchanged'. Suppose that the change in price depends on the last two observed outcomes. Is this an example of a Markov process? Explain your answer.
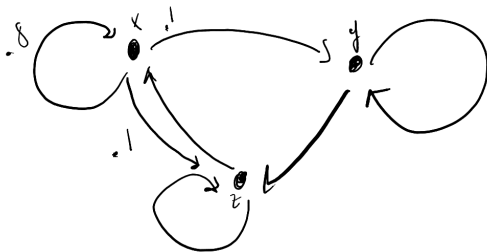
Example: Investment

- A particular utility stock is very stable, and in the short run, the probability that it increases or decreases in price depends only on the result of the preceding day's trading.

- The price of the stock is observed at 4 pm each day and is recorded as 'increased', 'decreased', or 'unchanged'.

- The sequence of observations forms a **Markov Process** because the current outcome (the price of stock) depends at most on the preceding outcome (price from yesterday).

Graphs

Definition
A **graph** is a diagram made of points (or vertices) and arrows that connect those points.

Example

Investment Example

- Suppose that the current state is 'increased'. That means that today the stock increased in price. The probability that tomorrow the price...
  - Increases: .3
  - Remains unchanged: .2
  - Decreases: .5
- Suppose that the current state is 'remains unchanged'. That means that today the stock didn't change price. The probability that tomorrow the price...
  - Increases: .6
  - Remains unchanged: .1
  - Decreases: .3
- Suppose that the current state is 'decreased'. That means that today the stock decreased in price. The probability that tomorrow the price...
  - Increases: .3
  - Remains unchanged: .4
  - Decreases: .3

Example: Sociology

- A sociologist postulates that the likelihood that in certain countries a woman will enter the labor force depends primarily on whether her mother worked.

- She designs an experiment to test this hypothesis by viewing the sequence of career choices of a woman, her daughters, her grand-daughters, her great-grand-daughters and so on.

Example: Women in the Labor Force

- Census studies from the 1960s reveal that in the US 80% of the daughters of women in the labor force also work outside of the home.
- 30% of the daughters of women not in the labor force work outside of the home. Assume that the trend remains unchanged from generation to generation.

Investment Example

- Suppose that the current state is 'increased'. That means that today the stock increased in price. The probability that tomorrow the price...
  - Increases: .3
  - Remains unchanged: .2
  - Decreases: .5
- Suppose that the current state is 'remains unchanged'. That means that today the stock didn't change price. The probability that tomorrow the price...
  - Increases: .6
  - Remains unchanged: .1
  - Decreases: .3
- Suppose that the current state is 'decreased'. That means that today the stock decreased in price. The probability that tomorrow the price...
  - Increases: .3
  - Remains unchanged: .4
  - Decreases: .3