

# Solution to Pascal's Identity Problem

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All of the 3-element subsets of  $\{1, 2, 3, 4, 5\}$  that contain 5.

$\{1, 2, 5\}$

$\{2, 3, 5\}$

$\{1, 3, 5\}$

$\{2, 4, 5\}$

$\{1, 4, 5\}$

$\{3, 4, 5\}$

Extra Explanation:

Because removing 5 gives you a 2-element subset of  $\{1, 2, 3, 4\}$ , and those are counted by  $C(4, 2)$ .

ex:  $\{1, 2, 5\} \longleftrightarrow \{1, 2\}$

$\vdots$

$\{3, 4, 5\} \longleftrightarrow \{3, 4\}$

# Solution to Router Through a City

1.  $C(5, 2) = 10$  Routes from a to c.

- Each route from a to c corresponds to a word of length 5, with letters S and E, with exactly 2 S's.

- Eg: Route: SSEEE  $\leftarrow$  length 5 word w/  
2 S's.

- Each ~~word~~ of these words corresponds to a 2-element subset of  $\{1, 2, 3, 4, 5\}$

Eg: SSEEE  $\longrightarrow$   $\{1, 2\}$

(look at the positions of your S's).

2. Solution (this is similar to some of the first multiplication-counting problems)

$$\frac{C(5, 2)}{\text{\# of ways to get from a to c}} \times \frac{C(3, 1)}{\text{\# of ways to get from c to d}} = 10 \times 3 = \boxed{30}$$