Conditional Probability

Section 6.4

March 31, 2015

Quiz

Four people are running for class president: Liz, Fred, Sue and Tom. The probability of Fred, Sue and Tom winning are .2, .35, and .15 respectively.

1. What is the probability that Liz will win?
2. What is the probability that a girl will win?
3. What is the probability that Fred will lose?

Conditional Probability

Definition

The **conditional probability** of the event E given F is computed by

$$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)}$$

Conditional Probability Example

- Suppose $Pr(E) = .6$, $Pr(F) = .3$, and $Pr(E \cap F) = .2$
  Calculate:

1. $Pr(E|F)$
2. $Pr(F|E)$
3. $Pr(E|F')$
4. $Pr(E'|F')$

Example

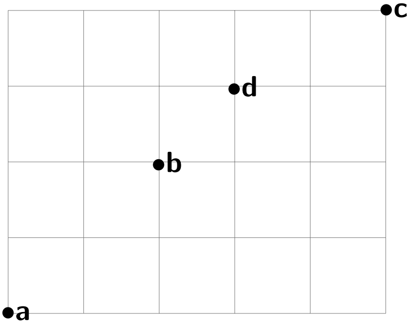
Suppose that we toss a coin three times and record the sequence of heads and tails. Let E be the event 'At most one head occurs' and F be the event 'both heads and tails occur'.

1. What is the probability of E?
2. What is the probability of F?

Example

Experiment

Choose a path from **a** to **c**, using only **E**ast and **N**orth steps. Assume that all paths are equally likely to occur.



Independence

Two events are **independent** of each other if the occurrence of one does not effect the likelihood that the other will occur.

Definition

In other words: The events E and F are **independent** if $Pr(E|F) = Pr(E)$.

- When E and F are independent, we have $Pr(E \cap F) = Pr(E) \cdot Pr(F)$.

Example

Suppose that E and F are two events.

- $Pr(E) = .4$
- $Pr(F) = .5$
- $Pr(E \cup F) = .7$
- Are E and F independent?

Example

Suppose that the probability of an event E is .4, the probability of an event F is .5, and the probability of the event $E \cap F$ is .2.

1. Draw a two circle Venn diagram, label one circle E, the other F, and fill in the appropriate probability into each of the 4 regions of the diagram.
2. Use the Venn diagram to compute $Pr(E|F')$.
3. Are the events E and F' independent? Explain your answer.