Selected solutions from chpt 5 project

1. $n=5,\ r=3$ and $n=5,\ r=2$

2. we want to show that

$$\frac{n!}{r!\,(n-r)!} = \frac{(n-1)!}{r!\,(n-1-r)!} + \frac{(n-1)!}{(r-1)!\,(n-1-(r-1))!}$$

Concrete example:

$n=6,\ r=4$

$$\frac{6!}{4!\,2!} = \frac{5!}{4!\,1!} + \frac{5!}{3!\,2!}$$

} get a common denominator on the RHS

$$\left[= \frac{5!}{(4\cdot 3\cdot 2\cdot 1)\cdot 1} + \frac{5!}{(3\cdot 2\cdot 1)(2\cdot 1)} \right]$$

multiply top + bottom by 4

multiply top and bottom by 2

$$= \frac{2\cdot 5!}{(4\cdot 3\cdot 2\cdot 1)(2\cdot 1)} + \frac{4\cdot 5!}{(4\cdot 3\cdot 2\cdot 1)(2\cdot 1)}$$

$$= \frac{5!\,(2+4)}{4!\,2!}$$

factor out 5! on top

#2 Continued:

Conclude: ~~SMI~~

we have $\dfrac{6 \cdot 5!}{4!\,2!} = \dfrac{6!}{4!\,2!}$ ✓

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3. Concrete example

$$C(6,3) = C(5,3) + C(5,2)$$

$C(6,3)$: counts 3-elmt subsets of $\{1,2,\ldots,6\}$

$C(5,3)$: counts 3 elmt subsets of $\{1,2,\ldots,5\}$

$C(5,2)$: counts 2 elmt subsets of $\{1,2,\ldots,5\}$

- Each 3 elemt subset of $\{1,2,3,4,5,6\}$ either contains 6 or it doesn't.
- If it doesn't contain 6, it's ~~really~~ also a 3-element subset of $\{1,2,3,4,5\}$
  - Counted by $C(5,3)$
- If it does contain 6, then taking 6 out we'll get a 2-elmt subset of $\{1,2,3,4,5\}$
  - Counted by $C(5,2)$

Extra Question

- Assuming $C(6,2) = C(6,4)$, use Pascal's Identity to show that $C(7,2) = C(7,5)$.

Solution:

- First we verify $C(6,1) = C(6,5) = 6$
  - $C(6,1)$ is # of 1-elmt subsets of $\{1,2,\dots,6\}$

$$\underbrace{\{1\},\ \{2\},\ \{3\},\ \{4\},\ \{5\},\ \{6\}}_{6 \text{ of these.}}$$

  - $C(6,5) = C(6,1)$ b/c every time we pick 1-elmt subset a 5-elmt subset, we leave a 5-elmt subset behind.

- eg:

$$\left.\begin{array}{ccc} \{1\} & \longleftrightarrow & \{2,3,4,5,6\} \\ \{2\} & \longleftrightarrow & \{1,3,4,5,6\} \\ \{3\} & \longleftrightarrow & \{1,2,4,5,6\} \\ \{4\} & \longleftrightarrow & \{1,2,3,5,6\} \\ \{5\} & \longleftrightarrow & \{1,2,3,4,6\} \\ \{6\} & \longleftrightarrow & \{1,2,3,4,5\} \end{array}\right\} \text{6 total}$$

Now use Pascal's Id:

$$C(7,5) = \underline{C(6,5)} + C(6,4) = \underline{C(6,1)} + C(6,2) = C(7,2)$$

equal

equal

A Student group is forming a committee w/ 3 members. Assuming there are 56 possible ways to choose the members of the committee, how many members does the student group have?

Solution: Use Pascal's Δ:

$n=0 \rightarrow 1$

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$n=8$ 1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

$\rightarrow$ 1 8 28 56 70 ~~70~~ 56 28 8 1

$r=0$ $r=1$ $r=2$

$C(8,3)$

$\therefore$ $n=8$ there are 8 members of the group.