Linear Programming Recap

Section 3.3 and Simplex Method

February 2, 2016

Steps for a solution

Theorem

The profit (minimized) function is maximized (minimized) at the vertices of the feasible set.

1. Write down the linear inequalities that define the feasible set and objective function
2. Graph the feasible set
3. Find vertices of the feasible set
4. Test each vertex in the objective function

Example

A clothing manufacturer makes dresses and jackets.

- Each dress requires 2 hours for cutting and each jacket requires 1 hour for cutting, but there are only 42 hours available
- Each dress requires 2 hours of sewing and each jacket requires 4 hours of sewing, but there are only 16 hours available
- Each dress requires 3 hours of finishing and each jacket requires 1 hour of finishing, but there are only 9 hours available
- The manufacturer earns a profit of \$3 for each dress and \$7 for each jacket it produces.

Vertices of feasible sets

Definition

A **vertex** of a feasible set is an intersection point of two boundary edges of the feasible set that is also feasible.

Warning

A feasible set need not be closed off (like a polygon).
It can be "open".

Where is the minimum?

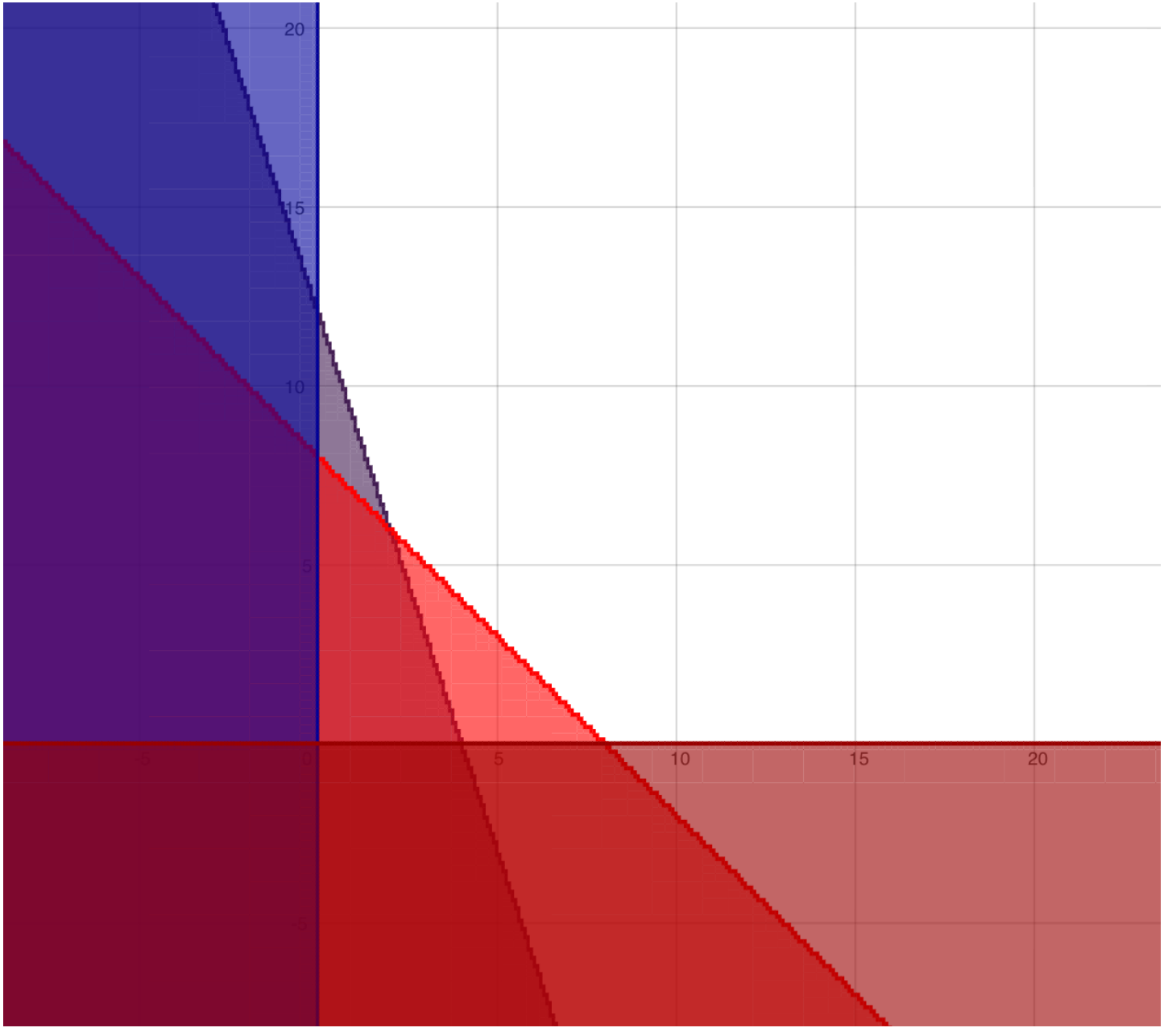
Consider the feasible set defined by the following linear inequalities:

$$\begin{cases} 3x + y \geqslant 12 \\ x + y \geqslant 8 \\ x \geqslant 0, y \geqslant 0 \end{cases}$$

What are the vertices?

Where is the objective function $5x + 4y$ minimized?

Graph of the feasible set



Linear programming for three variables

Suppose you want to compute a linear programming problem with the following constraints:

$$\begin{cases} x + y \leqslant 5 \\ x + y \geqslant 1 \\ x + z \leqslant 5 \\ x + z \geqslant 1 \\ y + z \leqslant 6 \\ y + z \geqslant 1 \end{cases}$$

How would we graph this?

We don't want to graph...

But we still need to find vertices. To find vertices in a three dimensional example we can use **Gauss Jordan elimination**!

Steps

1. Pick three constraints that have all three variables. This is a linear system!
2. Solve this system with Gauss Jordan elimination.
3. Check: Is the solution feasible? Not all intersection points are feasible!

Shipping Example: Three Variables

Suppose that a Michigan fruit farmer has stores in Chicago, IL and Grand Rapids, MI and farms in Holland, MI and Traverse City, MI. The cost of shipping a pound of apples from Holland to Chicago is \$6 and from Holland to Grand Rapids is \$3. The cost of shipping a pound of apples from Traverse City to Chicago is \$9 and from Traverse City to Grand Rapids is \$5.

Suppose that the Chicago store orders 25 pounds of apples, and the Grand Rapids store orders 30 pounds. Suppose further that the Holland farm has 45 pounds of apples and the Traverse City farm has 40 pounds.

What is the cheapest way to supply the orders for apples?