Solving Systems of Linear Equations

2.4: Matrix Inverses and Test 1 Review

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Matrix multiplication

Compute A^2 for

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

Back to linear systems

We can write a linear system as a matrix equation.

Example

What's the linear system we get when we do the matrix multiplication:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$$

To solve a system with matrix multiplication...

1. Write the system as a matrix equation: $AX = B$.
2. Find A^{-1}
3. Multiply both sides of the equation by A^{-1}:

$$A^{-1}AX = A^{-1}B$$

   Order matters!
4. Write the solution.

Example

$$\begin{cases} 4x + -2y + 3z = 0 \\ 8x - 3y + 5z = 1 \\ -7x - 2y + 4z = 0 \end{cases}$$

1. Write the system as a matrix equation $AX = B$.
2. Use the given matrix to find a solution for the system of linear equations.

$$A^{-1} = \begin{bmatrix} -2 & 2 & -1 \\ 3 & -5 & 4 \\ 5 & -6 & 4 \end{bmatrix}$$

How to find an inverse?

We will use Gauss-Jordan elimination to find the inverse of

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$$

1. Write your matrix next to the identity matrix:

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 5 & 0 & 1 \end{array}\right]$$

2. Perform elementary row operations until the **left** side is the identity matrix.
3. The **right** side is A^{-1}.

Example

Find the inverse for the matrix:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$