Sets and Inclusion-Exclusion

Section 5.1-5.3

October 13, 2015

Sets

Definition

A **set** is any collection of objects.
The objects belonging to a set are its **elements**. We use brackets to denote a set: $\{STUFF\}$.

- The set of odd numbers between 1 and 7: $\{1, 3, 5, 7\}$.
- The set of cats in my home: {Abby and Linus}
- The set that has no elements

- The set possible outcomes when tossing a coin twice: $\{HH, HT, TH, TT\}$
- The set of two letter words using the letters A and B: $\{AA, AB, BA, BB\}$

Key Property

The only thing we can say about a set S and an element x is whether or not x belongs to S.

IMPORTANT

A set does not see any distinction between its elements. No one element is better (or bigger).
The elements of a set are **not ordered**.

Example

Set	Not a set
$\{1, 2, 3, 4\}$	$(2, 1, 3, 4)$
{E,m,i,y,l}	Emily
{NCSU, Duke, UNC}	NCSU $>$ Duke $>$ UNC

What can we do...

We'd like to put sets together and take them apart.

Definition

1. The **union** of the sets S and T is $S \cup T$, the set of all elements that belong to S *or* T.
2. The **intersection** of the sets S and T is $S \cap T$, the set of all elements that belong to S *and* T.
3. A **subset** of S is a set T such that every element of T belongs to S. We write it as $T \subseteq S$.
4. Suppose $S \subseteq U$. The **complement** of S (relative to U) is the set of elements in U that do *not* belong to S.
   We write it as $U - S$ or S'.

Examples

1. Consider the following sets: $U = \{1, 3, 2, 6, a, e, i, o, u, y\}$, $S = \{2, 6, a, e, y\}$, $T = \{1, 3, 2, 6\}$, and $R = \{a, e, i, o, u\}$.
    1. Write down the elements set $S \cup T$.
    2. Write down the elements of the set $T \cap S'$.
    3. True or false: T is a subset of the complement of R.

Reality Check

True or False

1. The set S is a subset of $S \cup T$.
2. The set $S \cap T$ is a subset of S.
3. Suppose that $S \subseteq U$. The set S is a subset of $U - S$.

Reality Check

True or False

1. The set S is a subset of $S \cup T$.
2. The set $S \cap T$ is a subset of S.
3. Suppose that $S \subseteq U$. The set S is a subset of $U - S$.

Some Facts

- $S \cup T$ is the smallest set that contains the sets S and T. (It's bigger!)
- $S \cap T$ is the biggest subset of both S and T. (It's smaller!)

Counting

- The goal over the next few lectures will be to count elements of a set.
- We will use Venn diagrams to help us visualize the sets we're counting.

Simple Example

The two most common colors in the 200 flags of the member nations of the UN are red and white.

- 95 flags contain red
- 120 flags contain white
- 65 contain red and white

Question

How many flags contain exactly one color (just white or just red)?
How many flags don't contain red or white?

Let $n(S)$ denote the number of elements in S.

If we count the number elements in $S \cup T$, by $n(S) + n(T)$, then we'll over-count by exactly $n(S \cap T)$.

Inclusion-Exclusion Principle

$$n(S \cup T) = n(S) + n(T) - n(S \cap T)$$

Complicated Example

The three most common colors in the 200 flags of the member nations of the United Nations are red, white and blue. Draw a three circle Venn diagram with set U, the set of all flags of the member nations of the United Nations, set W for flags with white, R for the set of flags with red, and B for the set of flags with blue. Draw a three set Venn diagram and label the sets S, T, and W. Shade the appropriate set...

1. The set of flags that have all three colors
2. The set of flags that have blue and red, but not white
3. The set of flags that have blue and white but not red
4. The set of flags that have white, but not blue or red

Fill in the Venn Diagram

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- 96 flags contain red
- 111 flags contain white
- 85 flags contain blue
- 75 flags contain both red and white
- 60 flags contain both red and blue
- 70 flags contain both white and blue
- 50 flags have all three colors

Counting

1. How many flags have exactly two of the colors red, white and blue?
2. How many flags have red and blue?
3. How many flags do not contain any red, white, or blue?

Summary

- The 3-set Venn diagram has 8 regions
- Fill in the number of elements in each region.
- Start inside and work outward